Introduction to Differential Sedimentation

Differential Centrifugal Sedimentation, or DCS (sometimes also called "two-layer" sedimentation) is a widely used analysis method that produces extremely high resolution size distributions of microscopic to sub-microscopic particles. The normal measurement range for the method is from about 0.02 micron (20 nanometers) to about 30 microns (30,000 nanometers), though it is possible with some types of materials to extend the range to below 0.01 micron or to 50 microns or more. This document provides some background information on particle size analysis by sedimentation, explains how the DCS method works, describes the advantages and limitations of the method, and discusses likely future developments in DCS. Several example analyses are presented to help illustrate the capabilities of DCS.

Basic Theory of Particle Size Analysis by Sedimentation

Sedimentation of particles in a fluid has long been used to characterize particle size distribution. Stokes' law (1) is used to determine an unknown distribution of spherical particle sizes by measuring the time required for the particles to settle a known distance in a fluid of known viscosity and density. Sedimentation can be either gravitational (1 g-force), or centrifugal (many g-force).

Gravitational sedimentation is normally limited to particles of relatively large size, because the rate of sedimentation for small particles is too low to give a practical analysis time, and because Brownian motion of small particles becomes too large to allow effective settling. A very narrow distribution of small particles will be reported as a broad distribution when the rate of particle diffusion is comparable to the sedimentation rate. Very small particles (<0.1 micron) never settle by gravity unless they are extremely dense, so most types of very small particles can not be measured by gravitational sedimentation. Sedimentation in a centrifuge extends the range of sedimentation analysis to much smaller particles. High g-force makes sedimentation of small particles much faster than Brownian diffusion, even for very small particles. When a centrifuge is used, Stokes' law must be modified to account for the variation in g-force with distance from the center of rotation.

$D = ((18 \text{ c} \ln(R_f/R_o)) / ((\tilde{n}_p - \tilde{n}_f) \dot{u}^2)))^{1/2} / t^{1/2}$ (1)

Where $\, \boldsymbol{\varsigma} \,$ is the viscosity of the fluid

- $\tilde{\mathbf{n}}_{f}$ is the density of the fluid
- $\tilde{\mathbf{n}}_{P}$ is the density of the particle
- $\boldsymbol{R_o}$ is the starting radius of rotation
- R_f is the ending radius of rotation

- **ù** is the rotational speed (radians/second)
- t is arrival time at R_f (location of detector)

For a centrifuge running at constant speed and temperature, all of the parameters except time are constant during an analysis. The values for these are either well known or can be accurately measured. Within a broad range of analysis conditions, the modified form of Stokes' law accurately measures the diameter of spherical particles based on arrival time at the detector.

Methods of Sedimentation Analysis

There are two common sedimentation methods: integral, and differential. The following discussion explains the differences between these methods.

Integral Sedimentation

The integral method (Figure 1) is the oldest of the sedimentation methods. A detector



Figure 1 - Integral Sedimentation Method

beam (a light beam or x-ray beam) passes through the fluid at a known distance form the fluid surface, and measures particle concentration. The initial intensity of light or X-rays reaching the detector is a minimum, corresponding to the maximum concentration of particles. As particles settle through the fluid, the concentration of particles remaining in the dispersion falls, and the intensity of light or X-rays that reaches the detector increases. Stokes's law is used to calculate the size of particles that sediment out of the fluid as a function of time, and a particle size distribution is generated by plotting the measured concentration of particles against the calculated particle diameter. The result of the analysis is an integral representation of the particle size distribution. The method is called integral sedimentation because the sum (the "integral") of all particles smaller than a particular size is being continuously measured during the analysis. A differential particle size distribution can be generated from the integral results by applying mathematical differentiation with respect to diameter.

Integral sedimentation can be applied to particles lower in density than the fluid in which they are suspended. In this case, the particles have a net buoyancy, so they sediment toward the surface of the fluid rather toward the bottom.

There are three significant operational problems with integral sedimentation in a centrifuge. First, the initial conditions of the analysis are difficult to characterize. If the sample is added to a centrifuge that is already spinning, then there will be turbulent mixing of the sample dispersion as it is added to the centrifuge, which makes accurate measurement of sedimentation time difficult. If a sample is added to a centrifuge that is not spinning, and is later accelerated to high speed, then it is necessary to accurately measure and account for the changing speed during the acceleration period. It is also necessary to use a centrifuge of a design that insures there is no mixing of the sample during acceleration. Second, convection currents can develop during an analysis unless the temperature of the sample is held constant; any convection currents in the fluid can reduce both resolution and the accuracy of results. High speed centrifuges generate frictional heat, which makes it more difficult to maintain constant temperature in the sample fluid. Third, the sedimentation chamber must be emptied and cleaned following each sample, which increases operator labor.

Differential Sedimentation

Differential sedimentation (see Figure 2) was first reported in 1930 (2). A sample of particles to be analyzed is placed on top of a column of clear liquid at the start of the analysis, and particles settle according to Stokes' Law, just as in integral sedimentation. The detector initially reads maximum intensity, but the signal is reduced when particles reach the detector beam. The reduction in intensity indicates the concentration of particles in the detector beam. When an X-ray beam is used, the reduction in intensity is proportional to particle concentration. When a monochromatic light source is used, Mie theory light scattering can be applied to the intensity data to calculate particle concentration.

When all particles have passed the detector, the signal returns to the original level. A plot of the particle concentration against the calculated particle diameter produces a differential distribution. At any time during the analysis, only particles of one particular size range are being measured by the detector beam; all larger particles have already passed the beam, and all smaller particles have not yet arrived. The method is called differential sedimentation because only a tiny part of the distribution (a "differential") is

being measured by the detector beam at any time. An integral distribution can be generated from a differential distribution by applying mathematical integration with respect to particle diameter. A differential size distribution and its corresponding



Figure 2 - Differential Sedimentation Method

integral distribution are shown in Figure 3.

Actually running a differential sedimentation is a little more complicated than suggested by the above description. When a sample of particles which are more dense than the fluid in the column is placed on top of the column, the particles do not settle individually according to Stokes' Law. Instead, the entire sample suspension rapidly settles as a bulk fluid through the liquid column, in exactly the same way as a homogeneous liquid of higher density (like 10% sodium chloride in water) would settle through a column of another liquid of lower density (like water). The bulk settling of a sample in differential sedimentation is commonly called "streaming" or "sedimentation instability" (*3*). All information about the particle size distribution can be lost when streaming takes place. Several methods (*4,5,6*) have been developed to eliminate streaming. Each of these methods is effective because a slight density gradient is formed within the fluid column, prior to starting analyses. A wide range of fluids can be used to form a density gradient. In aqueous systems, gradually changing concentrations of methanol, ethanol, glycerine, sucrose, and many other materials have been used. In nonaqueous systems, many mixtures of fluids of different density can be used.

A density gradient eliminates streaming because at all times during the analysis the net density of the fluid, which is the average density of fluid plus any suspended particles,

increases continuously from top to bottom in the fluid column. The condition which guarantees stable sedimentation is given by Equation 2.

$$\frac{\ddot{a}\tilde{n}}{\ddot{a}x} \ge 0 \tag{2}$$

where \widetilde{n} is the net density of the fluid (particles plus fluid) x is distance from the surface of the fluid

When a small volume of a particle suspension is placed on the surface of the fluid



Figure 3 - Differential and Integral Distributions

column, the net density of the suspension is very slightly higher than the pure fluid; but the fluid just under the surface is also slightly higher in density than the pure fluid, due to the density gradient. There is no driving force for bulk settling of the particle suspension, so there is no instability, and the particles sediment through the fluid according to Stokes' Law. The required steepness of the density gradient depends upon the net density of the sample to be measured. A sample with higher net density (higher particle concentration and/or higher particle density) requires a steeper density gradient than a sample with lower net density. Most samples are diluted to low concentration, so only a very slight density gradient is required to insure stability. Density gradients of less than 0.01 g/ml per centimeter of fluid height are normally sufficient to insure complete stability. A density gradient also eliminates thermal convection, so sedimentation is not disrupted by slight changes in fluid temperature during an analysis. Relatively large temperature changes (>0.5°C) can cause some loss of accuracy unless they are accounted for, because fluid viscosity changes with temperature.

Differential Centrifugal Sedimentation

DCS Instrument Design

The most common design for DCS instruments is a hollow, optically clear disc that is driven by a variable speed motor. A typical disc cross section is shown in Figure 4. The disc can be of virtually any size, but manufacturers have settled on a diameter of approximately 125 to 150 mm. The detector beam is usually monochromatic light of relatively short wavelength (400 nm - 450 nm); though some instruments use a longer wavelength (~650 nm), or X-rays. Shorter wavelength light gives better detector sensitivity when particles smaller than 200nm are measured.



Figure 4 - Hollow Disc Centrifuge Design

To prepare the instrument for analysis, the disc is set in motion at constant speed, and then the disc chamber is filled with a fluid which contains a slight density gradient. Samples are prepared for analysis by dilution in a fluid of slightly lower density than the least dense fluid in the disc. The lower density fluid used for the sample reduces initial mixing of the fluid inside the disc with the sample. When a sample is injected (normally using a small syringe), it strikes the back inside face of the disc, and forms a thin film, which spreads as it accelerates radially toward the surface of the fluid. When the sample dispersion reaches the fluid surface, it quickly spreads over the surface, because it is of lower density (it "floats" on the higher density fluid). Once a sample is on the fluid surface, sedimentation of individual particles begins. The injection of a sample is rapid (typically <50 milliseconds), so the starting time for an analysis is well defined, and the precision of sedimentation time is quite good.

When an analysis is complete, the instrument is ready for the next sample. There is no need to empty and clean the centrifuge, so many samples can be run in sequence without stopping the centrifuge. The only limitation on continuous run time is that the density gradient slowly degrades due to molecular diffusion. When the density gradient is no longer steep enough to maintain stable sedimentation, the instrument must be stopped, emptied, and a new gradient formed. Typical gradient lifetime is 3 to 15 hours, depending on the molecular weight and viscosities of the materials that form the gradient.

Advantages and Limitations of the DCS Method

Accuracy and Repeatability

Accuracy and repeatability of the DCS method are very good in nearly all cases. Any significant inaccuracy in the results is caused by either inaccurate values for the physical parameters of the system (densities, viscosity, rotational speed, etc.), instability in the sedimentation, or by deviation of the sedimentation from Stokes' Law.

Physical Parameters

The overall accuracy of the analysis depends upon the combined accuracy of each of the values in Equation 1. For example, if the viscosity of the fluid is actually 2% higher than entered in Equation 1, then the reported particle size will be about 1% smaller than correct. It is possible to achieve nearly any desired level of accuracy by improving the accuracy of the parameters in Equation 1. An alternative method to improve accuracy is to use a narrow calibration standard of precisely known size to determine the effective combined value, K, for all the parameters in Equation 1. Equation 1 then reduces to:

 $D = K(1/t)^{0.5}$

Where K is a combination of constants t is time to reach the detector

(3)

A calibration standard can be used externally, where it is analyzed just before or just after an unknown sample to determine K, or internally, where a small amount of the calibration standard is added to the unknown. Instrument software finds the calibration peak within the distribution of the unknown sample, and adjusts the value for K so that the calibration standard peak is exactly the correct diameter. The adjusted value for K is applied to the entire distribution, so the accuracy of the analysis improves. Internal calibration gives extremely high accuracy and repeatability: the peak sizes in replicate analyses of an unknown are usually within +/- 0.25% when an internal standard is

used.

Sedimentation Stability

Any instability (streaming) during an analysis reduces both accuracy and resolution. Streaming causes the reported size distribution to be larger than correct, because during streaming particles move toward the detector faster than they would in normal sedimentation. Streaming usually takes place near the beginning of an analysis, when the entire sample is contained in a thin fluid layer near the surface. A small amount of streaming will cause the sample to form a broad initial band, followed by normal sedimentation; the result is both lower resolution and larger than correct reported sizes.

Commercial DCS instruments are normally set up to operate under conditions that always yield stable sedimentation. However, to verify that the sedimentation is stable, a direct means of confirming stability is needed. Some DCS instruments are equipped with a strobe light which is synchronized with the rotation of the centrifuge. This allows direct visual observation of the stability of sedimentation. With experience, an operator can judge if there is any instability based upon the appearance of the sedimentation. Other instruments rely on a narrow calibration standard to verify stability. When a calibration standard is used (either internal or external), evaluation of sedimentation stability can be made automatic; the instrument software can compare the measured width and shape of the calibration standard peak with the known width and shape for that calibration standard. Any significant change in distribution width or shape indicates instability in the sedimentation.

Deviation from Stokes' Law

Stokes' law does not accurately describe the sedimentation process if the Reynolds number for the system becomes too high. The Reynolds number increases with larger particles, faster sedimentation rate, and lower fluid viscosity. Most sedimentation analyses are run at low Reynolds numbers (<0.02), where deviation from Stokes' law is less than 0.5%. For example, at a centrifuge speed of 10,000 RPM, analysis of acrylic latex particles of 3 microns (density 1.13 g/ml) in water, will produce a Reynolds number of ~0.007, and a deviation from Stokes' law of ~0.25%. In cases where the Reynolds number is higher, deviation from Stokes' law can be taken into account by the instrument software so that the reported particle size distribution is accurate, regardless of Reynolds number.

Resolution and Data Density

Compared to most other particle size analysis methods, DCS gives distributions that have excellent resolution. Calibration standards with very narrow distributions can be routinely resolved when the ratio of diameters is ~1.05 (see Figure 5), and partially separated when the ratio is as little as ~1.02. In this article, resolution is defined as the minimum ratio between the diameters (larger/smaller) of two perfectly narrow peaks which allows those peaks to be completely separated. Compared to most other analysis methods, the number of data points that form the DCS distribution is also high.



Figure 5 - Example of Resolving Power

A typical distribution covering a dynamic range of 25 may contain more than 1,000 individual data points, while the distributions from some other analysis methods may contain as few as 128 data points, but covering a much broader size range.

The theoretical resolution of the DCS method depends almost entirely on three factors: 1) the volume of the sample, 2) the width of the detector beam compared to the distance the particles sediment during the analysis, and 3) Brownian motion of the particles during the analysis.

Sample Volume

The sample volume controls the thickness of the initial band of sample particles; no band of particles arriving at the detector can ever be more narrow than the initial sample band. Even a sample that is perfectly uniform in size will arrive at the detector beam as a band slightly wider than the initial sample band. (see Figure 6) It is therefore normal to use relatively small sample volumes to improve resolution. In commercial instruments, a sample of 100 microliters forms an initial band of about 0.04 mm width, while the sedimentation distance is ~ 10 mm. The initial sample band width adds only about ~ 0.008 to the theoretical resolution.

Detector Beam Width

The width of the detector beam relative to sedimentation depth is the most important factor that controls resolution. A detector beam of 0.4 mm width (typical for a commercial instrument) adds ~0.02 to the theoretical resolution for the instrument if the sedimentation distance is 10 mm. (see Figure 6) Increasing the sedimentation



Figure 6 - Resolution of the DCS method

distance (for example, 20 mm instead of 10 mm) improves resolution, but increases analysis time by a factor of more than 2. Reducing the width of the detector beam improves resolution, but as the beam becomes more narrow the total light reaching the detector is reduced, so there is a reduction in the signal to noise ratio as resolution improves. If the light source intensity is increased, then a more narrow beam can be used, and instrument resolution can be improved while maintaining signal to noise ratio.

Brownian Motion

The effect of Brownian motion during the analysis is normally negligible for particles larger than about 0.2 micron, because the amount of Brownian motion is small compared to the distance the particles sediment and the width of the detector beam. For smaller particles, and especially for relatively long analysis times, Brownian motion does hurt resolution somewhat. Brownian motion causes a random diffusion of particles, so a narrow band of particles becomes wider with time. However, even though the band broadens, the measured average size remains unchanged.

Figure 7 shows a overlay comparison of two analyses of the same narrow 538 nm polystyrene latex, plotted with equal area under both curves. Both analyses were run using a water/sucrose density gradient (0% to 4% sucrose). The narrower of these analyses was run at 8,500 RPM centrifuge speed (~3.25 minutes analysis time to the

distribution mean), the wider was run at 3,500 RPM centrifuge speed (~21 minutes analysis time to the mean, an artificially long analysis for this particle size due to the slow rotational speed). The slight change in peak width in the slower analysis demonstrates the effect of Brownian motion; the reported standard deviation for the peak increased from 0.014µ to 0.016µ. For the longer analysis, the instrument resolution changed from ~1.03 to ~ 1.038.

The speed of Brownian motion increases as the particle size becomes smaller, approximately in proportion with the inverse square root of the particle diameter. For example, particles of 50 nm diffuse about 3.3 times faster than the 538 nm particles shown in Figure 7, so the effect of diffusion on instrument resolution would be comparably greater with a similar analysis time. Resolution can be improved by applying mathematical deconvolution methods to the initial particle size distribution data. By using deconvolution, most of the effects of detector beam width, sample band width, and Brownian motion can be removed from the data, yielding a distribution with even higher resolution. Of course, for most all measurements, the basic resolution of the DCS method is more than adequate, and deconvolution is not needed.

(For more information on the effects of Browninan motion, see the CPS Disc Centrifuge Operating Manual, and the CPS technical paper titled "Effects of Brownian Motion".)



Figure 7 - Band broadening due to Brownian motion during sedimentation

Sensitivity and Sample Size

The DCS method is very sensitive, especially in the size range of 0.1µ to 5µ diameter,

where the efficiency of light scattering is high. For larger and smaller diameters, sensitivity becomes gradually lower. Figure 8 shows an analysis of a low concentration calibration standard (polyvinylchloride latex, 0.573μ mode). The injected sample was 50 μ liters, and the particle weight in the injected sample was ~2 μ g. Even though the prepared sample dispersion had little visually perceptible turbidity, the particle concentration was sufficient to give a high accuracy size distribution, with relatively low noise. Broad distributions require more sample weight, but any sample with a total dry weight of 50 to 100 μ g usually produces a good distribution. Some other analysis methods require much larger sample sizes.



Figure 8 - Analysis of a low concentration sample: ~ 2µg total dry weight.

Speed of Analysis and Dynamic Range

Total analysis time depends on centrifuge speed, particle density, fluid density, fluid viscosity, minimum particle size, maximum particle size, and data collection rate. Different commercial instruments often have large differences in total analysis time for the same sample. A higher data collection rate (more data readings per second) allows a wider dynamic range to be measured in the same total analysis time, because larger (faster moving) particles can be measured more accurately. A higher maximum centrifuge speed reduces total analysis time for samples with very small particles. Dynamic size range has a very strong effect on total analysis time. Using a constant speed centrifuge and constant detector position, and measuring a dynamic size range of 25 (ratio of largest size to be measured to smallest size in the distribution), the total analysis time will normally range ~ 10 minutes to ~ 40 minutes, depending on the instrument. If the dynamic range is 50, then analysis time for most samples will be from ~ 40 to ~160 minutes, depending on instrument. If the dynamic range is relatively

narrow (<15) then most samples can be analyzed within ~ 4 to ~ 16 minutes.

The total analysis time for samples with very wide dynamic range can be reduced by either changing the position of the detector beam during the analysis (moving it gradually toward the fluid surface), or by increasing the speed of the centrifuge during the analysis. These approaches are comparable to the temperature program commonly used with gas chromatography to broaden dynamic range. Employing these techniques can expand the practical dynamic range for a single analysis to >1000. From an operational standpoint, changing the centrifuge speed is preferable to changing the detector position, because a moving detector reduces resolution for small particles, and because a moving detector requires that the centrifuge be stopped, cleaned, and restarted after each analysis. Figure 9 shows the results of an analysis where the centrifuge speed was increased from 1,200 RPM to 16,500 RPM over the first 6.4 minutes of the analysis. Distribution data was collected over the size range of 32µ to 0.06µ, or a dynamic range of 533. Total analysis time was 15.7 minutes. A similar analysis at fixed speed would require 40 hours of analysis time, where Brownian motion would cause severe loss of resolution for the small end of the distribution. Changing the centrifuge speed requires a centrifuge chamber of special design, to avoid disruption of the fluid density gradient during acceleration and deceleration...



Low Density and Neutral Buoyancy Particles

Figure 9

The most important historical limitation for differential centrifugal sedimentation has been the requirement that the particles to be measured be significantly higher in density than the fluid in the centrifuge. A minimum density difference of 0.05 g/ml is desirable for most samples, and a difference of 0.1 g/ml or more is better. Some aqueous dispersions, such as polymer latexes and oil emulsions, often have particle densities near or below 1 g/cc. It is possible to use a mixture of water and methanol or ethanol, which has a density lower than water, to measure some types of low density samples, but many are not compatible with the required alcohol concentration. Many low density dispersions have historically been impossible to measure using the DCS method.

A recent development (7) has eliminated the requirement of high particle density. The new low density technique (see Figure 10) uses a density gradient made from a fluid of higher density than the particles to be measured; the sample dispersion is deposited at the bottom of the centrifuge chamber rather than at the surface.



Figure 10 - Modified disc design for low density particles.

The sample is prepared for analysis by dispersion in a fluid that is slightly higher in density than the fluid at the bottom of the centrifuge chamber. The sample spreads along the bottom of the chamber when it is injected, due to its higher density. The analysis proceeds normally, but the particles float toward the fluid surface rather than sinking toward the bottom. Stokes' law continues to accurately describe the motion of the particles. For aqueous sample dispersions where the particles are significantly lower in density than water (for example, polybutadiene latex), a normal aqueous density gradient can be used. In cases where the particles are close to the density of water (0.95 g/ml to 1.05 g/ml), the centrifuge can be filled with a density gradient made from deuterium oxide (density 1.107 g/ml), so that the particles have the required buoyancy for analysis. All aqueous particle dispersions are compatible with a density

gradient formed from deuterium oxide.

Figure 11 shows the results of an analysis of three polystyrene latexes, run using the low density technique. The density gradient was prepared from sucrose and deuterium oxide (0% to 4% sucrose). The polystyrene samples were prepared for analysis by dispersion in a solution of 6% sucrose in deuterium oxide. The polystyrene





concentration in the prepared sample was ~0.2% by weight, and 25µL were injected. The 6% sucrose in the sample preparation increased the density of the prepared sample to higher than the 4% sucrose solution at the bottom of the centrifuge chamber.

Non-spherical Particles

The weight distribution reported by the DCS method is a "Stokes-equivalent" distribution: the weight distribution of spherical particles that would yield the reported distribution. The Stokes-equivalent distribution is equal to the true weight distribution only if the particles in the distribution are spherical. Particles with other geometries are reported somewhat smaller than their actual weight distribution. For particles that closely approximate spheres (for example eicosahedrons), the measured distribution will be very nearly correct, while geometries very different from spheres, like long thin rods, will be reported as significantly smaller than their actual weight distribution.

Cylindrical rods with an aspect ratio of ~ 2 (length/width) produce a reported weight distribution about 5% smaller than correct, while rods with an aspect ratio of ~ 3

produce a reported weight distribution about 10% smaller than correct. Particles with a disk shape, ~ 2 times wider than they are thick, are reported as about 6% smaller than correct. For all non-spherical particles, no matter what the geometry, the DCS method produces very consistent and repeatable results, even if those results are not exactly correct in absolute weight sense. The DCS method is commonly used for characterization and quality control with a wide range of inorganic pigments, fillers, and abrasives, even though the particles being measured are not spherical in shape.

For more information about accurate measurement of non-spherical particles, see the CPS technical paper titled "Measuring Non-Spherical Particles".

Future Trends

Developments in DCS over the next five years are probable in four areas.

- 1. Overall instrument sensitivity and dynamic signal range will continue to improve, due to higher analog S/N ratio, higher resolution analog to digital conversion, and improved (software based) noise filtration. Sensitivity and dynamic signal range will likely improve by at least a factor of 5. An analysis that today requires at least 1µg dry sample weight will probably require ~0.2µg within the next 5 years. Improved sensitivity will allow analysis of near trace quantities of particulate contaminates in liquids. Contamination of one particle size with another (for example, 0.40µ particles contaminating a sample of 0.50µ particles) should be detectable at less than 1 part in 5,000.
- 2. Instrument resolution will continue to improve, due to better detector beam optics and optimized data deconvolution. Routine resolution of particles that differ in size by <2% should be possible.
- 3. Single run dynamic range will likely increase as DCS instruments are commercialized with the option to ramp up speed during an analysis. Single run dynamic range should reach 1000 or more. For a typical sample, all particles between 0.04 micron and 30 microns will be measured in a single analysis requiring only ~20 minutes. Wide dynamic range will allow DCS instruments to be used in applications where only light scattering methods are in use today.
- 4. Completely automatic on-line DCS systems will be developed if there is sufficient need for high resolution on-line measurement of particle size distributions in the 0.02µ to 30µ size range. A self-contained automated system (similar to on-line gas chromatographs) could be attached to the product stream from a continuous or batch process, and automatically sample, measure, and report particle size distributions at almost any desired frequency.

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